Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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14 pages

2225–7111

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

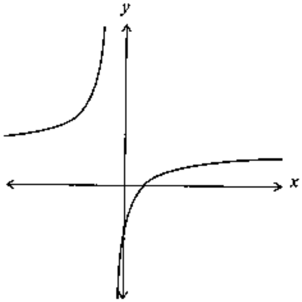
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

(This question continues on the following page)

2. [Maximum mark: 5]

The line L_1 is defined by the Cartesian equation $\frac{x-1}{2} = \frac{y+2}{3} = z$.

(a) Find a vector equation of L_1. [2]

A second line L_2 is defined by the vector equation $\boldsymbol{r} = \begin{pmatrix} 0 \\ 4 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, where $t \in \mathbb{R}$.

(b) Find the coordinates of the point where L_1 and L_2 intersect. [3]

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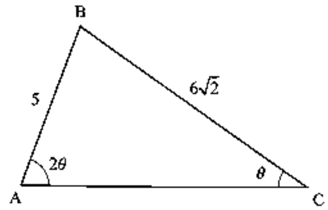
3. [Maximum mark: 7]

The following diagram shows a non-right angled triangle ABC.

diagram not to scale



$\text{AB} = 5$, $\text{BC} = 6\sqrt{2}$, $\text{A}\hat{\text{C}}\text{B} = \theta$ and $\text{B}\hat{\text{A}}\text{C} = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\theta = \frac{3\sqrt{2}}{5}$. [3]

(b) Hence, find $\sin\theta$. [2]

Point D is located on [AC] such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC. [2]

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4. [Maximum mark: 7]

Events A and B are such that $P(A \cup B) = \frac{5}{8}$ and $P(A \cap B') = \frac{7}{24}$.

(a) Find $P(B)$. [3]

(b) Given that events A and B are independent, find $P(A' | B)$. [4]

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5. [Maximum mark: 7]

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

(a) Find the possible values of k. [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

6. [Maximum mark: 7]

Consider the function $f(x)=\sqrt{x^2\ln x+4-x^2}$, where $x\in\mathbb{R}$, $x>0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l=\sqrt{x^2\ln x+4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin. [6]

7. [Maximum mark: 5]

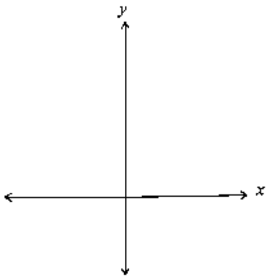
It is given that $x^4 + px^3 - 2x^2 + qx - 3$ is exactly divisible by $(x+1)^2$.

Find the value of p and the value of q, where $p, q \in \mathbb{R}$.

8. [Maximum mark: 8]

Consider the function $f(x) = \arccos x$ for $-1 \le x \le 1$.

(a) On the set of axes below sketch the graph of $y = f(x)$.
On your sketch clearly indicate the y-intercept and coordinates of the end points. [2]



(b) Solve $\arccos(x) + \arccos\left(x\sqrt{3}\right) = \frac{3\pi}{2}$, for $-\frac{1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$. [6]

9. [Maximum mark: 6]

Prove by contradiction that $\frac{1}{x(1-x)} \geq 4$ for $x \in \mathbb{R}$, $0 < x < 1$.

Do not write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

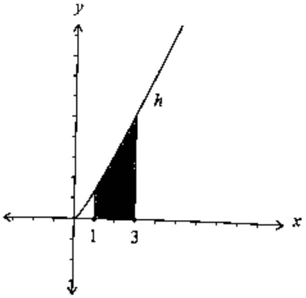
(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.



(d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q\ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

Do not write solutions on this page.

11. [Maximum mark: 17]

(a) Find the first four terms in the binomial expansion of $\sqrt{1+5x}$ in ascending powers of x. [4]

Consider the expression $(1+px)(1+qx)^{-1}$, where $p, q \in \mathbb{Q}$.

(b) Find the expansion of $(1+px)(1+qx)^{-1}$ in ascending powers of x, up to and including the term in x^2. [3]

The expansions found in parts (a) and (b) are identical up to the first three terms, for a value of p and a value of q.

(c) Show that $q = \frac{5}{4}$. [4]

(d) The expression $\frac{1+px}{1+qx}$, with $p = \frac{15}{4}$ and $q = \frac{5}{4}$, can be used as an approximation for $\sqrt{1+5x}$ where $|x| < \frac{1}{5}$.

(i) Hence, by finding a suitable value for x, find the approximation for $\sqrt{1.2}$ in the form $\frac{m}{n}$, where $m, n \in \mathbb{Z}$.

(ii) Now consider the approximation for $\frac{\sqrt{5}}{2}$. Explain why the approximation for $\frac{\sqrt{5}}{2}$ is not as accurate as the approximation for $\sqrt{1.2}$. [6]

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Do **not** write solutions on this page.

12. [Maximum mark: 19]

(a) Solve $z^2 = -1 - \sqrt{3}\,\mathrm{i}$, giving your answers in the form $z = r(\cos\theta + \mathrm{i}\sin\theta)$. [4]

Let z_1 and z_2 be the square roots of $-1 - \sqrt{3}\,\mathrm{i}$, where $\mathrm{Re}(z_1) > 0$.

Let z_3 and z_4 be the square roots of $-1 + \sqrt{3}\,\mathrm{i}$, where $\mathrm{Re}(z_3) > 0$.

(b) Expressing your answers in the form $z = a + b\mathrm{i}$, where $a, b \in \mathbb{R}$,

(i) find z_1 and z_2;

(ii) deduce z_3 and z_4. [4]

The four roots z_1, z_2, z_3 and z_4 are represented by the points A, B, C and D respectively on an Argand diagram.

(c) (i) Plot the points A, B, C and D on an Argand diagram.

(ii) Find the area of the polygon formed by these four points. [4]

The four roots z_1, z_2, z_3 and z_4 satisfy the equation $z^4 + 2z^2 + 4 = 0$.

The four roots $\frac{1}{z_1}$, $\frac{1}{z_2}$, $\frac{1}{z_3}$ and $\frac{1}{z_4}$ satisfy the equation $pw^4 + qw^2 + r = 0$ where $p, q, r \in \mathbb{Z}$.

(d) Find the value of p, q and r. [3]

The four roots $\frac{1}{z_1}$, $\frac{1}{z_2}$, $\frac{1}{z_3}$ and $\frac{1}{z_4}$ are represented by the points E, F, G and H respectively on an Argand diagram.

(e) (i) Find $\frac{1}{z_1}$ in the form $z = a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

(ii) Hence, deduce the area of the polygon formed by these four points. [4]